

Rules of indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ 2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<ol style="list-style-type: none"> 1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$
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Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Practice

1 Evaluate.

a 14^0

b 3^0

c 5^0

d x^0

2 Evaluate.

a $49^{\frac{1}{2}}$

b $64^{\frac{1}{3}}$

c $125^{\frac{1}{3}}$

d $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$

b $8^{\frac{5}{3}}$

c $49^{\frac{3}{2}}$

d $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2}

b 4^{-3}

c 2^{-5}

d 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$

b $27^{-\frac{2}{3}}$

c $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

f $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x .

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a	x^{-3}	b	x^0	c	$x^{\frac{1}{5}}$
d	$x^{\frac{2}{5}}$	e	$x^{-\frac{1}{2}}$	f	$x^{\frac{3}{4}}$

9 Write the following in the form ax^n .

a	$5\sqrt{x}$	b	$\frac{2}{x^3}$	c	$\frac{1}{3x^4}$
d	$\frac{2}{\sqrt{x}}$	e	$\frac{4}{\sqrt[3]{x}}$	f	3

Extend

10 Write as sums of powers of x .

a	$\frac{x^5 + 1}{x^2}$	b	$x^2\left(x + \frac{1}{x}\right)$	c	$x^{-4}\left(x^2 + \frac{1}{x^3}\right)$
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Answers

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| 1 | a | 1 | b | 1 | c | 1 | d | 1 |
| 2 | a | 7 | b | 4 | c | 5 | d | 2 |
| 3 | a | 125 | b | 32 | c | 343 | d | 8 |
| 4 | a | $\frac{1}{25}$ | b | $\frac{1}{64}$ | c | $\frac{1}{32}$ | d | $\frac{1}{36}$ |
| 5 | a | $\frac{3x^3}{2}$ | b | $5x^2$ | | | | |
| | c | $3x$ | d | $\frac{y}{2x^2}$ | | | | |
| | e | $y^{\frac{1}{2}}$ | f | c^{-3} | | | | |
| | g | $2x^6$ | h | x | | | | |
| 6 | a | $\frac{1}{2}$ | b | $\frac{1}{9}$ | c | $\frac{8}{3}$ | | |
| | d | $\frac{1}{4}$ | e | $\frac{4}{3}$ | f | $\frac{16}{9}$ | | |
| 7 | a | x^{-1} | b | x^{-7} | c | $x^{\frac{1}{4}}$ | | |
| | d | $x^{\frac{2}{5}}$ | e | $x^{-\frac{1}{3}}$ | f | $x^{\frac{2}{3}}$ | | |
| 8 | a | $\frac{1}{x^3}$ | b | 1 | c | $\sqrt[5]{x}$ | | |
| | d | $\sqrt[5]{x^2}$ | e | $\frac{1}{\sqrt{x}}$ | f | $\frac{1}{\sqrt[4]{x^3}}$ | | |
| 9 | a | $5x^{\frac{1}{2}}$ | b | $2x^{-3}$ | c | $\frac{1}{3}x^{-4}$ | | |
| | d | $2x^{-\frac{1}{2}}$ | e | $4x^{-\frac{1}{3}}$ | f | $3x^0$ | | |
| 10 | a | $x^3 + x^{-2}$ | b | $x^3 + x$ | c | $x^{-2} + x^{-7}$ | | |